How are groceries, images, and matrices related?

Joan Gómez Urgellés, PhD
Department of Mathematics
Polytechnic University of Catalonia EPSEVG
Vilanova i la Geltrú. Spain

joan.vicenc.gomez@upc.edu

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1. Introduction

This article is a proposition for the teaching / learning of some matrix calculation elements from mathematical modeling. As a matter of fact, some daily situations are established by having their matrices and their operations as a mathematical model as well, in particular by showing how we can create models to illustrate the matrix concept and also by introducing basic operations of difference and product of matrices.
Firstly, a matrix is shown as a mathematical model of an image and then how the matrix difference becomes a model for image comparison is discussed.
However, to do this task software such as Octave (or similar software) is necessary. This tool allows the research of a numerical model of a black and white image represented by a matrix.
Furthermore, we see how the matrix product is a model which can be naturally deduced from the grocery shopping routine. The main idea is to underline the matrix calculation epistemology in order to reinforce the students' cognitive character, bringing a contextual view of daily matters in real life at the same time, enriching the heuristic, thus allowing the visualization of the connection among the mathematical symbolism (introduced on the model) and the real situations.

1.1 Working with Images: The Matrices Difference as a Mathematical Model

1.1.1 A Matrix as a Mathematical Model of a Black and White image.

When we talk about images, mathematics has an important role. Actually, technically each image can be seen as a table of numbers (formally known as a matrix). Then, defining "an image composed by composition of M per N pixels”, it means that it can be represented by a matrix with M rows and N columns, generally with values between 0 and 255 (256 elements). The number of pixels is called "resolution". The procedure to obtain the matrix has been done through very sophisticated mathematical algorithms implemented by the software (or MatLab at a cost of 50 $ for students or 150 $ for home users).

When we say 15 per 13 pixels, we mean something similar to the figure below:

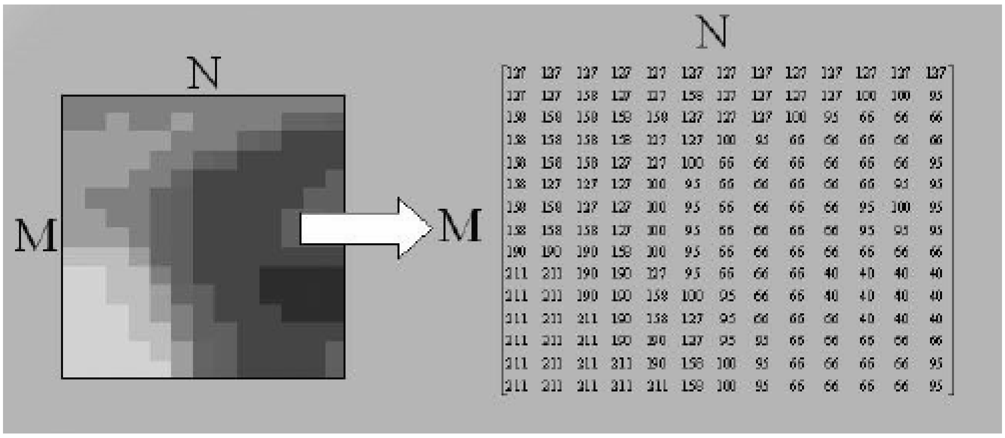


Figure 1 The pixels and their correspondent array

When we talk about “5 megapixels”, we are really talking about 5 million pixels. However, if we read “640 x 480”, that means 640 columns per 480 rows matrix.

Now we are analyzing a real situation. Consider these violin pictures:

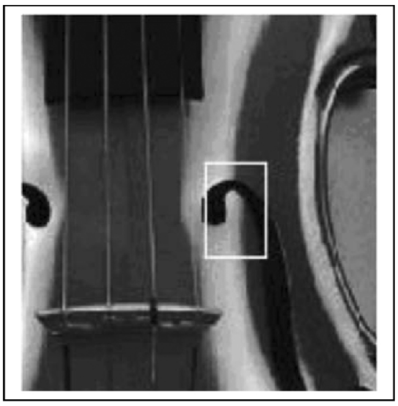


Figure 2 Violin Figure 3 Violin detail

This violin picture image matches the Matrix below:

170	170	180	181	186	189	191	186	179	172	164	152	98	56	57	53	50	55	50	44	45	41	41	45	46	47
168	175	177	177	188	194	187	186	179	169	161	153	93	59	55	49	50	49	47	43	44	46	44	45	43	40
169	171	182	183	190	194	192	187	172	171	158	136	90	62	53	55	52	48	49	42	45	48	43	40	43	41
175	175	185	192	195	194	193	187	174	172	164	150	87	55	52	51	48	47	48	47	46	42	40	43	43	43
176	181	185	189	191	192	195	190	178	174	165	145	85	54	48	47	46	48	49	49	42	45	41	42	43	43
178	180	189	194	197	197	196	189	181	174	164	146	73	53	58	48	46	45	45	50	46	46	43	41	43	43
182	186	192	197	200	201	198	190	184	184	176	156	74	46	48	45	45	39	41	42	43	44	42	40	41	43
182	189	195	200	209	217	215	205	190	165	142	116	79	49	45	43	43	42	46	47	43	42	41	41	39	44
183	187	200	211	236	223	140	56	28	16	11	7	12	13	23	38	49	48	46	46	45	41	40	38	40	43
191	191	220	214	143	43	9	5	7	7	8	6	8	7	7	12	23	33	44	43	44	42	42	40	42	43
196	227	190	45	5	6	7	9	8	7	6	7	4	9	7	6	7	11	24	47	37	39	41	44	43	41
215	182	25	4	8	8	9	7	9	6	7	46	44	7	8	8	6	8	9	19	31	38	40	42	45	41
180	21	6	8	9	8	9	9	2	25	122	178	164	39	9	7	9	7	7	8	15	30	38	38	41	38
63	2	8	8	7	9	8	4	22	142	203	179	163	126	50	8	6	8	7	9	6	16	33	38	39	41
11	8	9	8	8	8	6	31	141	183	187	181	168	167	112	35	9	8	8	7	9	8	14	31	40	40
5	9	8	8	7	8	9	117	172	169	187	182	163	158	131	65	21	7	7	7	6	8	8	18	37	37
8	9	8	8	9	7	6	62	174	172	183	180	165	149	127	85	33	12	8	7	8	8	8	9	21	37
10	7	9	8	8	7	7	12	134	181	181	174	160	153	128	88	41	26	8	8	8	7	7	7	8	27
9	8	8	8	7	7	7	7	68	182	176	170	164	154	119	92	41	35	17	8	7	8	7	7	7	14
9	9	7	8	8	7	7	7	21	152	176	168	160	151	126	87	41	39	29	8	6	7	7	8	7	8
9	8	9	8	8	7	8	7	10	117	182	166	150	147	120	75	39	39	35	16	6	8	7	6	7	6
10	9	9	7	8	8	7	8	8	108	177	167	159	151	105	82	42	39	37	27	8	8	7	7	7	8
10	9	8	8	7	8	8	8	11	122	172	160	152	152	117	80	49	42	37	33	13	6	8	7	8	7
10	10	9	10	8	8	8	6	26	146	168	160	150	144	110	82	44	38	39	34	18	8	7	8	7	8
7	10	10	9	9	9	8	6	97	167	157	161	149	147	102	66	49	42	36	34	25	7	8	6	8	8
35	9	7	9	8	7	4	42	151	161	162	160	153	150	105	77	43	40	35	35	30	9	7	8	7	6
115	50	24	12	13	26	66	142	157	155	163	160	152	148	107	67	48	40	35	34	31	12	9	7	7	8
140	123	117	95	92	129	154	153	158	159	162	157	151	148	105	62	44	38	38	34	32	17	7	8	6	8
136	134	145	146	143	152	151	157	166	167	166	160	153	147	112	78	43	39	41	36	33	23	7	7	9	7
138	142	144	144	150	153	156	158	168	171	171	157	149	155	108	61	51	43	40	37	34	24	8	7	6	9
143	141	145	149	153	159	159	157	167	176	170	163	160	154	110	62	48	43	38	35	32	30	12	8	8	7
148	150	145	151	156	159	158	162	169	172	167	162	156	153	114	59	53	45	38	35	30	33	17	7	7	8
150	150	150	157	164	164	166	170	174	172	174	166	155	151	116	71	51	47	41	36	34	33	21	8	9	9
145	148	153	158	160	168	168	171	178	178	170	165	161	153	104	66	47	43	40	37	34	33	21	8	8	8
151	151	156	157	163	166	165	174	176	177	170	165	163	154	118	67	45	41	40	35	33	34	25	9	7	8
147	152	156	161	167	169	167	171	177	180	180	167	154	155	128	80	52	38	37	37	34	34	26	10	7	8
154	153	158	164	170	169	171	172	185	181	173	171	159	156	128	81	60	43	42	40	36	33	27	10	8	8
150	155	162	167	168	172	172	176	179	180	175	169	163	155	133	77	54	44	44	38	38	33	29	11	8	8

Table 1 Matrix of the Violin

Take a glance at this unbelievable Numeric Table, even the density and the placement of the numbers “drawing” the violin profile. Next, the procedure in order to get the model is fully explained.

Actually, we can choose any image in our computer, by selecting with the cursor over the image, right clicking it, and then opening a Properties Window that shows all of the information about the image size. Depending on the resolution and the available space on the disk, it is possible to save the image in different formats such as BMP, TIFF, or JPEG as well.

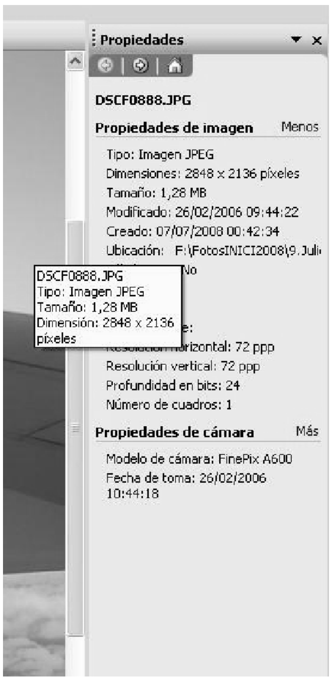


Figure 4 Octave Properties Window

In order to get a matrix model of an image, one needs software such as Octave. Formally Octave works with sophisticated numeric methods for obtaining the Matrix Model. The procedures are explained below.

1.1.2 Octave: Generating the Pixels Matrix of an image

Octave is a free program available for Windows, Mac, and Linux, developed for Numeric Calculations. It is available on the web https://www.gnu.org/software/octave/ It was developed around 1988, created by Chemical Engineering students from Texas University and Wisconsin-Madison University to be applied to support Chemical Reactors drawing. Actually, Octave is a free option to the well-known MathLab.
Octave has a wide kit of tools to solve algebra, calculation, and statistics problems. It is also able to process digital images. A Smartphone version is available.

We are working with Black and White images because the associated Matrix is bi-dimensional, which means that it is a Numbering Table with rows and columns. On the other hand, in case of colored images, a “three-dimensional Matrix” would be generated, and each color would be obtained from the basic RGB (Red, Green, Blue).

With Octave, we can generate the Pixel Matrix of any image.

How can we do that?

By following the steps below:

1.1.2.1 To install the program, go to the link:
https://www.gnu.org/software/octave/ or
ftp://ftp.gnu.org/gnu/octave/windows/octave-4.0.0_0-installer.exe

1.1.2.2 Once the Octave installation is completed, run the program by opening a similar window such as:

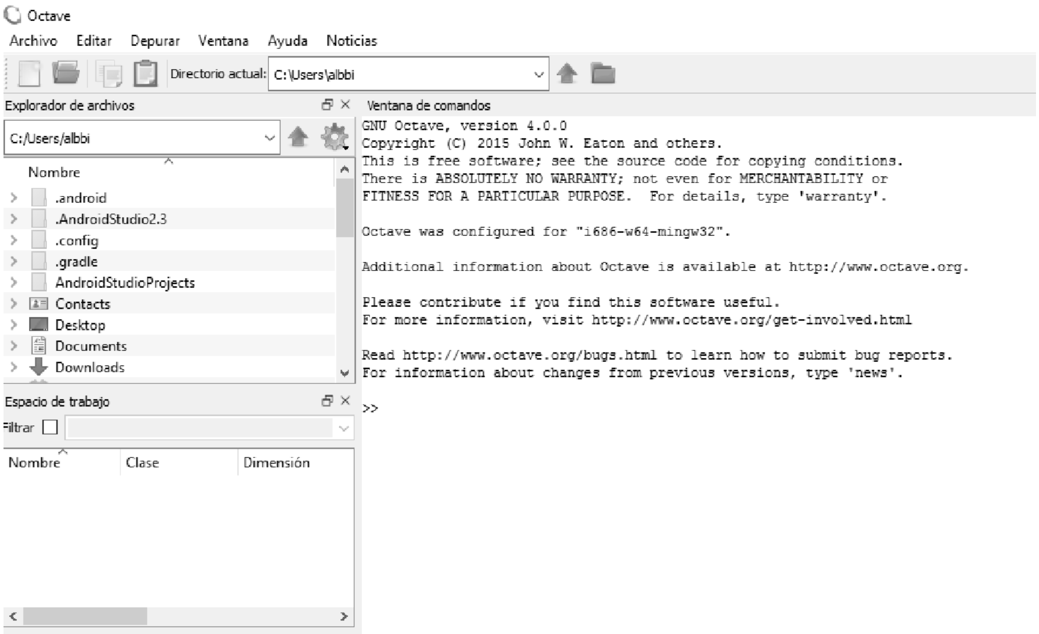


Figure 5 Running Octave

1.1.2.3Choose a previously saved image in the directory.

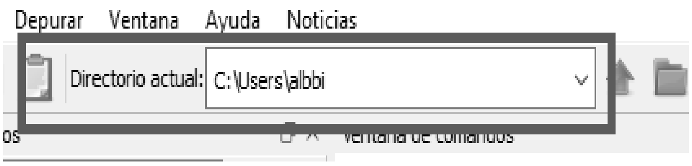


Figure 6 Saving in the directory

1.1.2.4 And then, select the image whose Pixel Matrix Associated we want to know.

Remember that it is necessary to save the image in the folder created by Octave, which in our case is ***C:\Users\albbi***.

Literally, in the Octave window we should go to the directory in which the images have been saved.

With the image selected in the folder, introduce the following instruction in the command line:

image = imread ('image name. extension')

SUMMARIZING: We can introduce the concept of Matrix as a model of a Black and White image.

2. Modeling Experience

2.1. Introduction

Now, we are doing an experience which has been explained in the classroom by showing the Matrix difference as a Mathematical Model of an image. In our example, we are considering two different black and which images, previously saved in our computer, and comparing both of them.

In our computer, these images have been saved as “imatge1.jpg “, “imatge2.jpg”

Figure 7. Imatge 1:

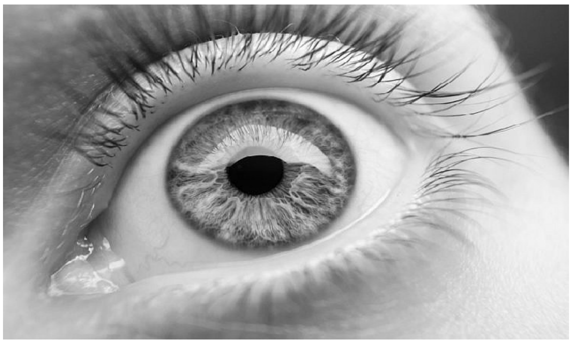
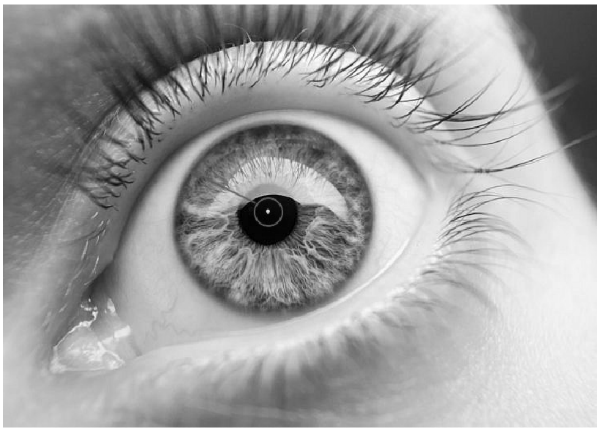


Figure 8. Imatge 2:



Apparently, these pictures seem identical, but this is not true; the second picture has a small different dot in yellow in order to be easily seen.

Now we are introducing the following lines in octave command:

Variable Name I = imread (“fotografia1.jpg”);

```
>> I = imread("fotografia1.jpg");
```

Figure 9. An Octave command line for the variable I

As shown above, “I” has been chosen as a Variable Name.

In this line, we are saving the Variable “I”, the “imatge 1”. However, we are really saving the Matrix of picture 1.

Next, we are introducing the command Variable Name 2= I2= imread (“fotografia2.jpg”);

```
>> I2 = imread("fotografia2.jpg");
```

Figure 10 An Octave command line for the variable I2

Saving the variable “I2” (“picture 2”)

By pressing “Enter” after each instruction, the window shows respectively the Matrix as a Mathematical Model of each image respectively obtaining:

Table 2 1st Image Partial view (imatge 1)

```
>> I
I =

 Columns 1 through 21:

  159  159  159  159  159  159  159  159  161  16
  161  161  161  161  161  161  161  161  159  15
  161  161  161  161  161  161  161  161  157  15
  159  159  159  159  159  159  159  159  157  15
  158  158  158  158  158  158  158  158  158  15
  158  158  158  158  158  158  158  158  158  15
  158  158  158  158  158  158  158  158  156  15
  157  157  157  157  157  157  157  157  154  15
  155  155  155  155  155  155  155  155  157  15
  155  155  155  155  155  155  155  155  155  15
  156  156  156  156  156  156  156  156  154  15
  155  155  155  155  155  155  155  155  153  15
  154  154  154  154  154  154  154  154  153  15
```

Table 3 Full Capture of the “imatge 1” Matrix

159	159	159	159	159	159	159	159	161	161	162	$\boxed{163}$	$\boxed{164}$	164	165	165	166	166	167	168	169
161	161	161	161	161	161	161	161	159	159	160	161	$\boxed{162}$	162	163	163	164	165	165	166	167
161	161	161	161	161	161	161	161	157	$\boxed{158}$	158	159	160	160	161	161	163	164	164	165	166
159	159	159	159	159	159	159	159	157	157	158	159	$\boxed{160}$	160	161	161	164	164	165	166	167
158	158	158	158	158	158	158	158	158	158	159	$\boxed{159}$	160	161	162	162	165	165	166	167	168
158	158	158	158	158	158	158	158	158	158	159	159	160	161	$\boxed{161}$	162	164	164	165	166	167
158	158	158	158	158	158	158	158	156	156	157	$\boxed{157}$	158	159	160	160	161	162	162	163	165
157	157	157	157	157	157	157	157	154	154	155	$\boxed{155}$	$\boxed{156}$	157	158	158	159	159	160	161	162
155	155	155	155	155	155	155	155	157	157	158	158	159	159	159	160	164	164	164	164	164
155	155	155	155	155	155	155	155	155	156	156	156	157	157	158	158	162	162	162	162	162
156	156	156	156	156	156	156	156	154	154	154	155	155	156	156	157	159	159	159	159	159
155	155	155	155	155	155	155	155	153	153	154	154	155	156	156	156	156	$\boxed{156}$	157	157	157
154	154	154	154	154	154	154	154	153	153	154	155	155	156	156	157	155	155	155	156	156
152	152	152	152	152	152	152	152	151	152	152	153	154	155	155	155	153	$\boxed{154}$	154	154	155
150	150	150	150	150	150	150	150	148	148	149	150	151	151	152	152	151	151	$\boxed{152}$	152	$\boxed{152}$
148	148	148	148	148	148	148	148	145	146	146	147	148	149	149	150	149	$\boxed{150}$	150	150	151
147	147	147	148	148	149	149	149	148	147	147	147	147	147	147	148	148	148	148	149	149
146	147	147	147	148	148	149	149	147	146	146	145	145	146	146	147	148	148	148	149	149
146	146	146	147	147	147	147	148	146	145	145	144	144	145	145	146	147	147	148	148	149
146	146	146	146	146	146	147	147	146	145	145	145	145	145	145	146	146	146	147	147	148
146	146	146	146	146	146	146	146	147	146	146	145	145	146	146	147	145	145	146	146	147
147	147	147	147	147	147	147	147	147	147	146	146	146	146	147	147	145	145	146	146	147
148	148	148	148	147	147	147	147	146	146	145	145	145	145	146	146	146	146	147	147	148
149	149	148	148	148	148	148	147	145	144	144	143	143	144	144	145	147	147	148	148	149
149	148	148	147	146	145	145	145	143	143	143	143	143	143	143	143	146	146	146	147	147
149	149	148	147	146	146	145	145	143	143	143	143	143	143	143	143	145	146	146	147	147
149	149	148	147	146	146	145	145	143	143	143	143	143	143	143	143	145	145	146	146	147
148	148	148	147	146	145	145	144	143	143	143	143	143	143	143	142	145	145	145	146	146

Analogically for the second image:

Table 4 2nd Image Partial view (imatge 2)

```
>> I2
I2 =

ans(:,:,1) =

 Columns 1 through 18:

  159  159  159  159  159  159  159  159  161  161
  161  161  161  161  161  161  161  161  159  159
  161  161  161  161  161  161  161  161  157  157
  159  159  159  159  159  159  159  159  157  157
  158  158  158  158  158  158  158  158  158  158
  158  158  158  158  158  158  158  158  158  158
  158  158  158  158  158  158  158  158  156  156
  157  157  157  157  157  157  157  157  154  154
  155  155  155  155  155  155  155  155  157  157
  155  155  155  155  155  155  155  155  155  156
  156  156  156  156  156  156  156  156  154  154
  155  155  155  155  155  155  155  155  153  153
  154  154  154  154  154  154  154  154  153  153
  152  152  152  152  152  152  152  152  151  152
```

159	159	159	159	159	159	159	159	161	161	162	162	163	164	165	165	166	166	167	168	169
161	161	161	161	161	161	161	161	159	159	160	161	161	162	163	163	164	165	165	166	167
161	161	161	161	161	161	161	161	157	157	158	159	160	160	161	161	163	164	164	165	166
159	159	159	159	159	159	159	159	157	157	158	159	159	160	161	161	164	164	165	166	167
158	158	158	158	158	158	158	158	158	158	159	160	160	161	162	162	165	165	166	167	168
158	158	158	158	158	158	158	158	158	158	159	159	160	161	162	162	164	164	165	166	167
158	158	158	158	158	158	158	158	156	156	157	158	158	159	160	160	161	162	162	163	165
157	157	157	157	157	157	157	157	154	154	155	156	157	157	158	158	159	159	160	161	162
155	155	155	155	155	155	155	155	157	157	158	158	159	159	159	160	164	164	164	164	164
155	155	155	155	155	155	155	155	155	156	156	156	157	157	158	158	162	162	162	162	162
156	156	156	156	156	156	156	156	154	154	154	155	155	156	156	157	159	159	159	159	159
155	155	155	155	155	155	155	155	153	153	154	154	155	156	156	156	156	157	157	157	157
154	154	154	154	154	154	154	154	153	153	154	155	155	156	156	157	155	155	155	156	156
152	152	152	152	152	152	152	152	151	152	152	153	154	155	155	155	153	153	154	154	155
150	150	150	150	150	150	150	150	148	148	149	150	151	151	152	152	151	151	151	152	153
148	148	148	148	148	148	148	148	145	146	146	147	148	149	149	150	149	149	150	150	151
147	147	147	148	148	149	149	149	148	147	147	147	147	147	147	148	148	148	148	149	149
146	147	147	147	148	148	149	149	147	146	146	145	145	146	146	147	148	148	148	149	149
146	146	146	147	147	147	147	148	146	145	145	144	144	145	145	146	147	147	148	148	149
146	146	146	146	146	146	147	147	146	145	145	145	145	145	145	146	146	146	147	147	148
146	146	146	146	146	146	146	146	147	146	146	145	145	146	146	147	145	145	146	146	147
147	147	147	147	147	147	147	147	147	147	146	146	146	146	147	147	145	145	146	146	147
148	148	148	148	147	147	147	147	146	146	145	145	145	145	146	146	146	146	147	147	148
149	149	148	148	148	148	148	147	145	144	144	143	143	144	144	145	147	147	148	148	149
149	148	148	147	146	145	145	145	143	143	143	143	143	143	143	143	146	146	146	147	147
149	149	148	147	146	146	145	145	143	143	143	143	143	143	143	143	145	146	146	147	147
149	149	148	147	146	146	145	145	143	143	143	143	143	143	143	143	145	145	146	146	147
148	148	148	147	146	145	145	144	143	143	143	143	143	143	143	142	145	145	145	146	146

Table 5 Full Capture of the 2nd Image Matrix

We realize that the difference between the images reflects in the different Matrix Model values. Strictly Speaking: we can get the differences between images by subtracting these matrices and concluding that the regions with zeros do not have changes. On the other hand, the regions with values different from zero mean that they do have changes.

SUMMARIZING: The difference between Matrices would be a Mathematical Modeling to compare images.

The Model has been applied in the class to the First Course of Computer Science EPSEVG University, as a group work developed by students, despite the fact that they had never worked with Matrices before. However, they were able to explain the work in the classroom to their other classmates, as shown in the picture below.

Figure 11   Computer Science University class

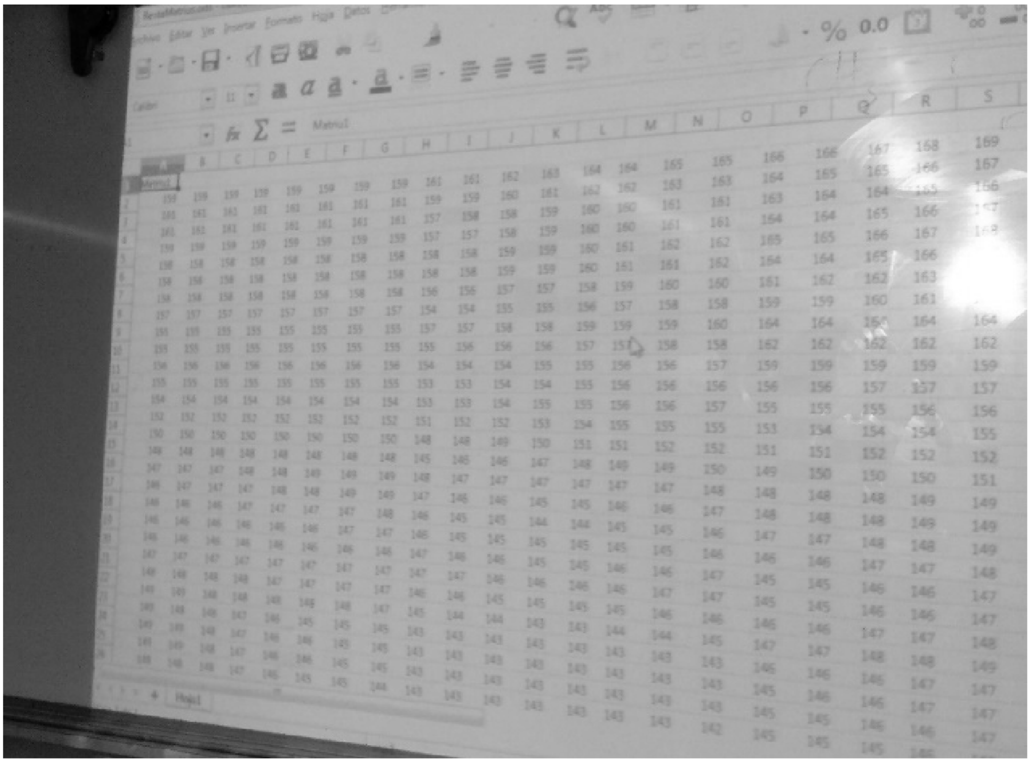


Figure 12 The Matrix obtained by the Computer Science students

Octave also allows to subtract the obtained Matrices. It's even possible to select the Matrices by attaching them to the Excel database and then subtracting them.

In our experience:

Table 6 Matrix 1

1	Matriu1																				
2	159	159	159	159	159	159	159	159	161	161	162	163	164	164	165	165	166	166	167	168	169
3	161	161	161	161	161	161	161	161	159	159	160	161	162	162	163	163	164	165	165	166	167
4	161	161	161	161	161	161	161	161	157	158	158	159	160	160	161	161	163	164	164	165	166
5	159	159	159	159	159	159	159	159	157	157	158	159	160	160	161	161	164	164	165	166	167
6	158	158	158	158	158	158	158	158	158	158	159	159	160	161	162	162	165	165	166	167	168
7	158	158	158	158	158	158	158	158	158	158	159	159	160	161	161	162	164	164	165	166	167
8	158	158	158	158	158	158	158	158	156	156	157	157	158	159	160	160	161	162	162	163	165
9	157	157	157	157	157	157	157	157	154	154	155	155	156	157	158	158	159	159	160	161	162
10	155	155	155	155	155	155	155	155	157	157	158	158	159	159	159	160	164	164	164	164	164
11	155	155	155	155	155	155	155	155	155	156	156	156	157	157	158	158	162	162	162	162	162
12	156	156	156	156	156	156	156	156	154	154	154	155	155	156	156	157	159	159	159	159	159
13	155	155	155	155	155	155	155	155	153	153	154	154	155	156	156	156	156	156	157	157	157
14	154	154	154	154	154	154	154	154	153	153	154	155	155	156	156	157	155	155	155	156	156
15	152	152	152	152	152	152	152	152	151	152	152	153	154	155	155	155	153	154	154	154	155
16	150	150	150	150	150	150	150	150	148	148	149	150	151	151	152	152	151	151	152	152	152
17	148	148	148	148	148	148	148	148	145	146	146	147	148	149	149	150	149	150	150	150	151
18	147	147	147	148	148	149	149	149	148	147	147	147	147	147	147	148	148	148	148	149	149
19	146	147	147	147	148	148	149	149	147	146	146	145	145	146	146	147	148	148	148	149	149
20	146	146	146	147	147	147	147	148	146	145	145	144	144	145	145	146	147	147	148	148	149
21	146	146	146	146	146	146	147	147	146	145	145	145	145	145	145	146	146	146	147	147	148
22	146	146	146	146	146	146	146	146	147	146	146	145	145	146	146	147	145	145	146	146	147
23	147	147	147	147	147	147	147	147	147	147	146	146	146	146	147	147	145	145	146	146	147
24	148	148	148	148	147	147	147	147	146	146	145	145	145	145	146	146	146	146	147	147	148
25	149	149	148	148	148	148	148	147	145	144	144	143	143	144	144	145	147	147	148	148	149
26	149	148	148	147	146	145	145	145	143	143	143	143	143	143	143	143	146	146	146	147	147
27	149	149	148	147	146	146	145	145	143	143	143	143	143	143	143	143	145	146	146	147	147
28	149	149	148	147	146	146	145	145	143	143	143	143	143	143	143	143	145	145	146	146	147
29	148	148	148	147	146	145	145	144	143	143	143	143	143	143	143	142	145	145	145	146	146

Table 7 Matrix 2

32	Matriu2																				
33	159	159	159	159	159	159	159	159	161	161	162	162	163	164	165	165	166	166	167	168	169
34	161	161	161	151	161	161	161	161	159	159	160	161	161	162	163	163	164	165	165	166	167
35	161	161	161	151	161	161	161	161	157	157	158	159	160	160	161	161	163	164	164	165	166
36	159	159	159	159	159	159	159	159	157	157	158	159	159	160	161	161	164	164	165	166	167
37	158	158	158	158	158	158	158	158	158	158	159	160	160	161	162	162	165	165	166	167	168
38	158	158	158	158	158	158	158	158	158	158	159	159	160	161	162	162	164	164	165	166	167
39	158	158	158	158	158	158	158	158	156	156	157	158	158	159	160	160	161	162	162	163	165
40	157	157	157	157	157	157	157	157	154	154	155	156	157	157	158	158	159	159	160	161	162
41	155	155	155	155	155	155	155	155	157	157	158	158	159	159	159	160	164	164	164	164	164
42	155	155	155	155	155	155	155	155	155	156	156	156	157	157	158	158	162	162	162	162	162
43	156	156	156	156	156	155	156	156	154	154	154	155	155	156	156	157	159	159	159	159	159
44	155	155	155	155	155	155	155	155	153	153	154	154	155	156	156	156	156	157	157	157	157
45	154	154	154	154	154	154	154	154	153	153	154	155	155	156	156	157	155	155	155	156	156
46	152	152	152	152	152	152	152	152	151	152	152	153	154	155	155	155	153	153	154	154	155
47	150	150	150	150	150	150	150	150	148	148	149	150	151	151	152	152	151	151	151	152	153
48	148	148	148	148	148	148	148	148	145	146	146	147	148	149	149	150	149	149	150	150	151
49	147	147	147	148	148	149	149	149	148	147	147	147	147	147	147	148	148	148	148	149	149
50	146	147	147	147	148	148	149	149	147	146	146	145	145	146	146	147	148	148	148	149	149
51	146	146	146	147	147	147	147	148	146	145	145	144	144	145	145	146	147	147	148	148	149
52	146	146	146	146	146	145	147	147	146	145	145	145	145	145	145	146	146	146	147	147	148
53	146	146	146	146	146	145	146	146	147	146	146	145	145	146	146	147	145	145	146	146	147
54	147	147	147	147	147	147	147	147	147	147	146	146	146	146	147	147	145	145	146	146	147
55	148	148	148	148	147	147	147	147	146	146	145	145	145	145	146	146	146	146	147	147	148
56	149	149	148	148	148	148	148	147	145	144	144	143	143	144	144	145	147	147	148	148	149
57	149	148	148	147	146	145	145	145	143	143	143	143	143	143	143	143	146	146	146	147	147
58	149	149	148	147	146	145	145	145	143	143	143	143	143	143	143	143	145	146	146	147	147
59	149	149	148	147	146	145	145	145	143	143	143	143	143	143	143	143	145	145	146	146	147
60	148	148	148	147	146	145	145	144	143	143	143	143	143	143	143	142	145	145	145	146	146

The Matrix difference is:

64	Matriu1-Matriu2:																				
65	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
67	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
69	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
71	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1
80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
84	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
85	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
88	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
89	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 8 Result of the difference between Matrix 1 and Matrix 2

The resultant Matrix clearly shows the regions of the image in which all differences have been observed.

2.2 Another example has been gathered from the Written Press. It refers to finding / spotting the differences.

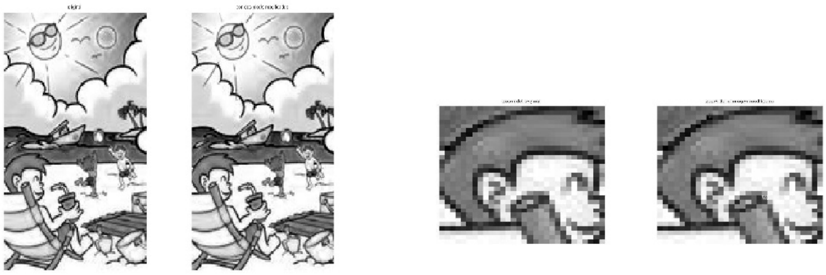


Figure 13 The Game of Differences

In detail, we realized that as a model, they have respectively their Numerical Matrix:

Table 9 Searching the differences 1st Matrix

105	113	98	103	112	99	99	108	113	115	139	116	74	52	29	38	37	43	54	68	72	70
103	105	92	99	111	106	111	123	128	91	46	32	56	91	114	121	130	128	126	122	122	123
88	86	97	108	99	115	124	77	22	44	72	100	121	130	132	129	128	128	130	131	129	128
98	92	114	96	135	137	27	6	71	94	115	125	125	125	132	140	134	136	137	138	139	138
96	102	86	130	113	19	26	86	118	125	129	128	126	125	128	131	133	131	130	130	130	132
96	99	120	119	23	11	118	135	122	129	133	139	142	138	129	122	128	129	129	129	132	136
108	129	132	28	30	126	118	118	149	151	149	141	133	127	126	125	113	115	118	123	127	129
87	107	38	26	103	117	115	163	141	147	147	142	131	123	118	117	121	118	117	118	123	126
59	0	20	94	110	121	149	131	151	149	145	140	134	129	125	119	91	81	73	66	66	73
237	60	65	111	123	119	135	136	144	122	149	130	117	132	111	47	20	19	25	41	40	32
133	54	114	126	123	127	115	133	132	120	128	130	126	121	35	50	139	193	213	204	199	212
64	90	143	129	118	129	122	132	130	128	112	119	115	46	54	210	240	249	239	247	252	245
76	142	135	126	118	124	133	94	75	77	114	134	112	28	181	236	249	255	244	245	255	252
103	144	120	127	121	127	97	45	44	22	22	49	107	28	198	250	236	255	254	253	254	252
127	122	129	127	120	137	54	65	180	225	176	53	31	42	204	248	255	255	254	255	254	253
130	117	131	123	119	132	34	136	248	212	244	219	57	47	233	236	234	255	255	245	243	255
110	123	117	121	125	114	29	177	231	112	108	226	144	70	233	243	247	231	237	253	243	242
78	126	121	128	115	124	46	177	247	233	148	92	215	168	239	247	244	249	245	234	234	255
41	117	115	115	132	127	35	167	250	170	72	107	243	236	255	214	147	123	120	159	254	243
108	51	130	120	127	130	66	94	255	93	195	214	228	253	163	78	79	93	47	49	243	250
223	37	68	128	118	115	108	33	195	147	120	243	239	154	36	138	191	94	52	66	176	255

Table 10 Searching the differences 2nd Matrix

107	98	120	93	111	91	116	92	102	107	92	105	119	123	120	78	66	49	30	31	41	48	48
105	113	98	103	112	99	99	108	113	115	139	116	74	52	29	38	37	43	54	68	72	70	71
103	105	92	99	111	106	111	123	128	[illegible]	46	32	56	91	114	121	130	128	126	122	122	123	124
88	86	97	108	99	115	124	77	22	44	72	100	121	130	132	129	128	128	130	131	129	128	124
98	92	114	96	135	137	27	6	71	94	115	125	125	125	132	140	134	136	137	138	139	138	137
96	102	86	130	113	19	26	86	118	125	129	128	126	125	128	131	133	131	130	130	130	132	133
96	99	120	119	23	11	118	135	122	129	133	139	142	138	129	122	128	129	129	129	132	136	138
108	129	132	28	30	126	118	118	149	151	149	141	133	127	126	125	113	115	118	123	127	129	131
87	107	38	26	103	117	115	163	141	147	147	142	131	123	118	117	121	118	117	118	123	126	131
59	0	20	94	110	121	149	131	151	149	145	140	134	129	125	119	91	81	73	66	66	73	84
237	60	65	111	123	119	135	136	144	122	149	130	117	132	111	47	20	19	25	41	40	32	38
133	54	114	126	123	127	115	133	132	120	128	130	126	121	35	50	139	193	213	204	199	212	216
64	90	143	129	118	129	122	132	130	128	112	119	115	46	54	210	240	249	239	247	252	245	248
76	142	135	126	118	124	133	94	75	77	114	134	112	28	181	236	249	255	244	245	255	252	248
103	144	120	127	121	127	97	45	44	22	22	49	107	28	198	250	236	255	254	253	254	252	255
127	122	129	127	120	137	54	65	180	225	176	53	31	42	204	248	255	255	254	255	254	253	253
130	117	131	123	119	132	34	136	248	212	244	219	57	47	233	236	234	255	255	245	243	255	230
110	123	117	121	125	114	29	177	231	112	108	226	144	70	233	243	247	231	237	253	243	242	221
78	126	121	128	115	124	46	177	247	233	148	92	215	168	239	247	244	249	245	234	234	255	189
41	117	115	115	132	127	35	167	250	170	72	107	243	236	255	214	147	123	120	159	254	243	213
108	51	130	120	127	130	66	94	255	93	195	214	228	253	163	78	79	93	47	240	243	250	240
223	37	68	128	118	115	108	33	195	147	120	243	239	154	36	138	191	94	52	66	176	255	255

As previously shown, the differences between images have been found in row 3, column 10. Over there, number 91 has been converted to 2. Also in row 21, in column 20 number 49 has been converted to 240.
This means that by subtracting both matrices, zero is obtained in all operations except in row 3, column 10, and in row 21, column 20 as well.

To our students, we can comment on another daily example when the Difference between Matrices reaches a remarkable role in the security field.

2.3 The difference between matrices as a Model of Security System.

According to our previous results, we can compare Images. Think about a hypothetical frame series (in black and white) captured by surveillance camera inside a bank. Now, considering the models of two consecutive images, the "intelligent camera" processes the difference between two Matrices. If Matrix “zero” (null) is obtained, we realize that no movement has been made, in this case it is not necessary to record all associated images to those matrices. On the other hand, all different matrices will be recorded registering the movement inside the bank.
This simplified example explains how the "intelligent surveillance cameras" do an enterprise night surveillance.

3 Shopping at a Supermarket: A Model of the Product between Matrices

In the following situation, the students, naturally discovering how to make the product of matrices into a Model, link the quantities of buying products at a Supermarket to their prices with total expenses.
Now we will do an easy study in order to clarify which of the two supermarkets is the least expensive merchant. We are shopping twice a week, buying the articles and quantities according to the table below:

Table 11 A Daily Consumption

	Pork Loin (kg)	Oranges (kg)	Lettuce (3 units / tray)
1st Day	1	3	1
2nd Day	3	2	2

This table can be written in a different way:

$$Q = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \end{pmatrix}.$$

And then for each supermarket we calculate:

- What are the expenses for the first day?
- What are the expenses for the second day?

We are also doing a global calculation:

- What is the least expensive option for each day?

These questions could be proposed to our students so that they can calculate and achieve their own conclusions.

First supermarket:

Figure 14 Supermarket 1 Logo.

Figure 15, 16 and 17 Expenses at Supermarket 1

We are writing the price list in the table below:

Table 12 Articles and Prices in Supermarket 1

Alcampo	Pork Loin (kg)	Oranges (kg)	Lettuce (3 units / tray)
Price	6,49 €	1,25 €	0,75 €

Second supermarket

Figure 18 2nd Supermarket Logo

Figures 19, 20, 21 Expenses at Supermarket 2

Repeating the same procedure with Carrefour price list:

Table 13 Articles and Prices in Supermarket 2

Carrefour	Pork Loin (kg)	Oranges (kg)	Lettuce (3 units / tray)
Price	5,90 €	0,85 €	1,25 €

Daily Calculation of our expenses in each supermarket:

Alcampo

1st Day → $1 \cdot 6{,}49 + 3 \cdot 1{,}25 + 1 \cdot 0{,}75 = 10{,}99$ €

2nd Day → $3 \cdot 6{,}49 + 2 \cdot 1{,}25 + 2 \cdot 0{,}75 = 23{,}47$ €

Carrefour

1st Day → $1 \cdot 5{,}90 + 3 \cdot 0{,}85 + 1 \cdot 1{,}25 = 9{,}70$ €

2nd Day → $3 \cdot 5{,}90 + 2 \cdot 0{,}85 + 2 \cdot 1{,}25 = 21{,}90$ €

Mathematically, we can write the expenses in each supermarket as:

Alcampo Supermarket

1st Day → $(1,3,1) \cdot (6'49, 1'25, 0'75) = 10'99$ €

2nd Day → $(3,2,2) \cdot (6'49, 1'25, 0'75) = 23'47$ €

Carrefour Supermarket

1st Day → $(1,3,1) \cdot (5'90, 0'85, 1'25) = 9'70$ €

2nd Day → $(3,2,2) \cdot (5'90, 0'85, 1'25) = 21'90$ €

The applied procedure is the well-known "***scaled euclidean product".*** It is remarkable that the students had been building the scaled product spontaneously.

SUMMARIZING: A daily matter such as shopping at a Supermarket has a Mathematical Model, the *"scaled euclidean product"*.

Globally, we can write the expenses in each supermarket as:

Alcampo Supermarket

1st and 2nd Day

$$\begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 6{,}49 \\ 1{,}25 \\ 0{,}75 \end{pmatrix} = \begin{pmatrix} 10{,}99 \\ 23{,}47 \end{pmatrix}$$

Carrefour Supermarket

1st and 2nd Day

$$\begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 5{,}90 \\ 0{,}85 \\ 1{,}25 \end{pmatrix} = \begin{pmatrix} 9{,}70 \\ 21{,}90 \end{pmatrix}$$

Here we are naturally building the ***product of a matrix by a vector column.***

We can join the tables in order to allow a better situation view and the Matrix Model:

Q: Quantity of Products

Table 14 Articles and Quantities

	Pork Loin (kg)	Oranges (kg)	Lettuce (3 units / tray)

1st Day	1	3	1
2nd Day	3	2	2

$$Q=\begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \end{pmatrix}.$$

P: Price in each supermarket

Table 15 Articles and Prices Supermarket 1

Alcampo	Pork Loin (kg)	Orange s(kg)	Lettuce (3 units / tray)
Price	6,49 €	1,25 €	0,75 €

Table 16 Articles and Prices Supermarket 2

Carrefour	Pork Loin (kg)	Oranges (kg)	Lettuce (3 units / tray)
Price	5,90 €	0,85 €	1,25 €

$$P=\begin{pmatrix} 6{,}49 & 5{,}90 \\ 1{,}25 & 0{,}85 \\ 0'75 & 1{,}25 \end{pmatrix}$$

D: Expenses Table

Table 17 Daily Expenses in both Supermarkets

	Alcampo	Carrefour
Expenses 1st Day	10,90 €	9,70 €
Expenses 2nd Day	23,47 €	21,90 €

$$D=\begin{pmatrix} 10{,}90 & 9{,}70 \\ 23{,}47 & 21{,}90 \end{pmatrix}$$

By doing it as previously mentioned, it is possible to introduce the ***Product*** *of Matrix.* The following Mathematical Model links Quantities (Q), Price (P), and Expenses (D):

$$\boxed{Q \cdot P = D}$$

SUMMARIZING: We realize that by linking purchased quantities and prices, it is possible to naturally obtain the algorithm to multiply Matrices.

Going back to the previous supermarket comparison, Carrefour Supermarket Chain has the best deals. The total expenses in Carrefour mean significant money saving when compared with those in Alcampo Supermarket.

The students who did this exercise discovered how to multiply Matrices naturally. Now, the professor feels free to propose situations by introducing the inverse matrix concept and other elements of Matrix calculation.

These examples are useful to show how the use of real life situations makes it feasible to find patterns (models) giving information about the proposed situations.

Obviously, the reader can translate the involvement of the Mathematical Modeling in the Academic Curricula, at the same time realizing that competent teaching is taking place.